Closing Wed: HW_3A, 3B (6.1, 6.2)
Closing Mon, Jan 30: HW_3C (6.3)
Visit office hours 1:30-3:00pm in PDL C-339

Exam 1 is Thursday, Jan 26th in your normal quiz section. It covers 4.9, 5.1-5.5, 6.1, 6.2
Allowed:

- One 8.5 by 11 inch sheet of ***handwritten*** notes (front and back)
- A Ti-30x IIs calculator (this model only!)
- Pen or pencil (no red or green)
- No make-up exams.

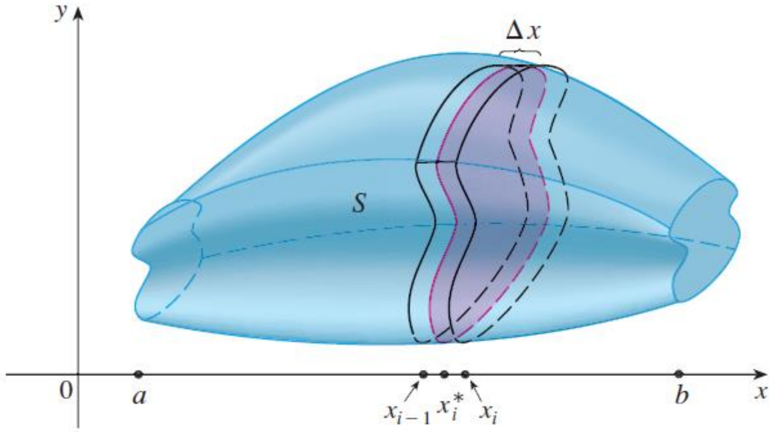
All homework is fair game. Expect problems like the homework. Know the concepts well. Practice on old exams.

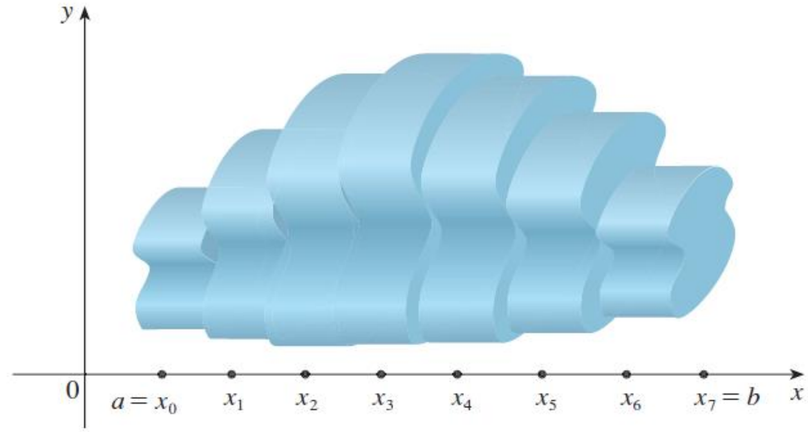
Quick Review:

4.9 Antiderivatives (tomato problems)
5.1/2: Riemann sums, Integral Notation
5.3: Fundamental Theorem of Calculus
5.4: Net/Total Change (abs. values)
5.5: Substitution (several problems)
6.1: Area between curves
6.2: Volumes by cross-sectional slicing

Please bring old exam and/or old homework questions to lecture on Wednesday.

6.2 Finding Volumes Using Cross-Sectional Slicing





If we can find the general formula, $A(x_i)$, for the area of a cross-sectional slice, then we can approximate volume by:

$$\text{Volume of one slice} \approx A(x_i)\,\Delta x$$

$$\text{Total Volume} \approx \sum_{i=1}^{n} A(x_i)\,\Delta x$$

This approximation gets better and better with more subdivisions, so we say

$$\text{Exact Volume} = \lim_{n\to\infty} \sum_{i=1}^{n} A(x_i)\,\Delta x$$

And we conclude

$$\text{Volume} = \int_a^b A(x)dx$$

$$= \int_a^b \text{"Cross-sectional area formula"}dx$$

Volume using cross-sectional slicing

1. Draw! Cut **perpendicular** to the axis of rotation.
   Note: If you draw a line at the cut, the *axis you cut across is the variable you are using*!

2. Draw a typical cross-section, label Δx or Δy and label *x* or *y*, appropriately.

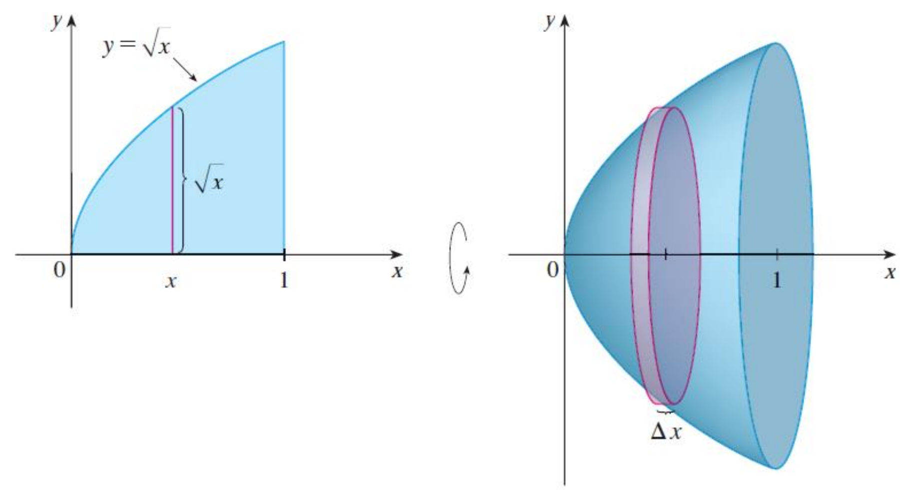
3. Label **everything** in terms of the appropriate variable.

4. Area? Find the formula for the area of a cross-sectional slice.

Disc: Area = π(radius)^2
Washer: Area = π(outer)^2 - π(inner)^2
Square: Area = (Height)(Length)
Triangle: Area = ½ (Height)(Length)

5. Integrate the area formula.

Example: Consider the region, *R*, bounded by $y = \sqrt{x}$, y = 0, and x = 1. Find the volume of the solid obtained by rotating R about the x-axis.

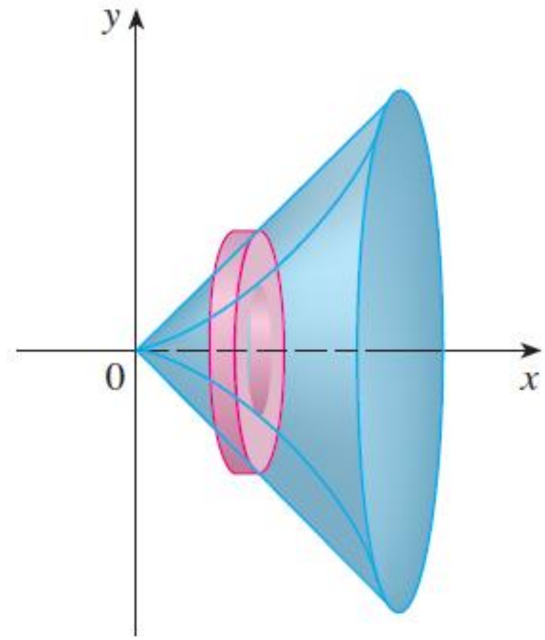


(a) (b)

Example: Consider the region, R, bounded by $y = \sqrt{x}$, x = 0, and y = 1. Find the volume of the solid obtained by rotating R about the y-axis.

Example: Consider the region, R, bounded by $y = x$ and $y = x^4$.
Find the volume of the solid obtained by rotating R about the x-axis.

1. Draw and label!
2. Cross-sectional area?
3. Integrate area.



Example: Consider the region, R, bounded by $y = x$ and $y = x^4$.
(R is the same as the last example).

(a) Now rotate about the horizontal line y = -5. What changes?

(b) Now rotate about the horizontal line y = 10. What changes?

Example:

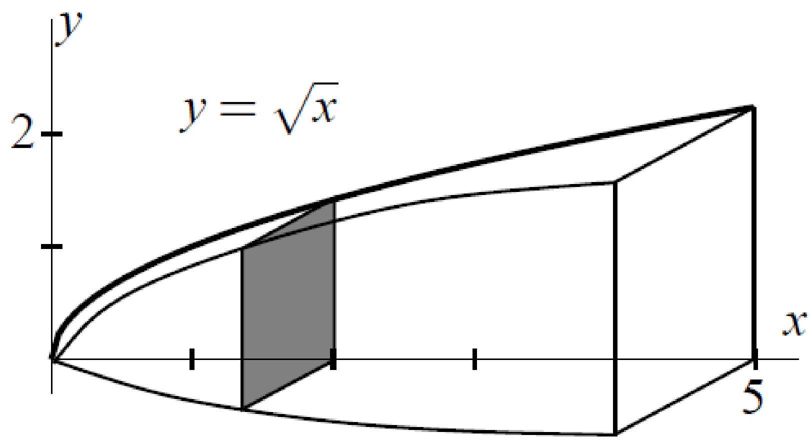
Set up an integral for find the volume obtained by rotating the region bounded by $y = x^3$, $y = 8$, and $x = 0$ about the vertical line $x = -10$.

Example:

(From an old final and homework)

Find the volume of the solid shown.

The cross-sections are squares.



1. Draw and label!
2. Cross-sectional area?
3. Integrate area.

Summary (Cross-sectional slicing):

1. Draw Label
2. Cross-sectional area?
3. Integrate area.

This method has one major limitation:
If the cross-sections are perpendicular to the x-axis, then you must use dx.

If the cross-sections are perpendicular to the y-axis, then you must use dy.

What if we were rotating about the x-axis and we wanted to use dy (or around y-axis and we want to use dx)?
Cross-sectional slicing wouldn’t work!
We need another method. That is what we will do in 6.3.